Observations of Magnetomorphic Oscillations in Zn and Cd by a Modified Field-Modulation Technique

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Large-amplitude magnetomorphic oscillations in the radial Hall effect produced by the interaction of modulation-field-induced eddy currents with a dc component of magnetic field have been observed. Oscillations due to the third-zone lens were observed in both cadmium and zinc. Hole-arm oscillations were also detected in cadmium. This technique allows the detection of signals with a sensitivity an order of magnitude greater than has been possible with previous techniques.

INTRODUCTION

The kinetic coefficients of electron transport in metals are known to have oscillatory components sinusoidal in the magnetic field when one of the sample dimensions is comparable with the mean free path.[1] These oscillations arise as a result of diffuse boundary scattering which limits the spiraling trajectory along the magnetic field. A group of electrons having sufficiently similar properties to give rise to the effect exists when the Fermi surface is such that (1) there is a limiting point where $\mathbf{v}=\nabla_p\epsilon$ is parallel to $\mathbf{H}$, or (2) the rate of change of cross-sectional area with respect to momentum displacement along $\mathbf{H}$, $\partial S/\partial p_3$, has an extremal value. Cadmium has a lens-shaped electron surface in the third zone which exhibits oscillations of type (1) for $\mathbf{H}\|[0001]$ and exhibits oscillations of type (2) for $\mathbf{H}$ in the (0001) plane. In addition, oscillations of type (2) have been observed for $\mathbf{H}\|[0001]$ due to the "hole arms" in the second zone. The period of the oscillations P_0 is given by

$$P_0=2\pi gc(\cos\theta)/ea\,, \tag{1}$$

where θ is the angle that the magnetic field makes with the normal to a thin-plate sample of thickness a, c is the velocity of light, e is the electronic charge, and g is the extremal value of $(1/2\pi)\partial S/\partial p_3$ for type-(2) oscillations, which reduces to the Gaussian radius of curvature (momentum units) of the limiting point for type-(1) oscillations.[1,2] The two types of oscillations in cadmium have been studied both by dc techniques in which the oscillatory component is observed superimposed on the gross magnetoresistivity and Hall resistivity,[3] and also by conventional field-modulation methods where dc current is passed through the sample in the presence of a modulated magnetic field giving rise to an ac signal containing harmonics of the modulation frequency.[4] These harmonics are proportional to successive derivatives of the magnetoresistance. The latter method allows direct observation of the oscillatory component. This paper reports the observation of very strong magnetomorphic oscillations in the radial Hall effect produced by the interaction of modulation-field-induced eddy currents with the steady component of magnetic field. The system is self-differentiating such that the harmonic content of the radial Hall effect measures various derivatives of the magnetomorphic oscillation. The method is applied both to Cd and to Zn.

[1] C. G. Grenier, K. R. Efferson, and J. M. Reynolds, Phys. Rev. **143**, 406 (1966).
[2] V. L. Gurevich, Zh. Eksperim. i Teor. Fiz. **35**, 668 (1958) [Soviet Phys. JETP **8**, 464 (1959)].
[3] H. J. Mackey, J. R. Sybert, and J. T. Fielder, Phys. Rev. **157**, 578 (1967).
[4] P. D. Hambourger, J. A. Marcus, and J. A. Munarin, Phys. Letters **25A**, 461 (1967).

EXPERIMENTAL DETAILS

X-ray–oriented single-crystal samples were spark machined from 69 purity ingots on a Servo-Met spark machine. The plane of each sample was parallel to (0001) to within 1 deg. The samples were not chemically etched or polished so that the planed surfaces were characteristic of a range-6 finish. Two cadmium samples were prepared; one was a rectangular parallelepiped $1.57\times16.7\times8.5$ mm. The other was a disk of thickness 1.88 mm and 17.0 mm in diam. The single zinc sample was a disk of 17.1 mm diam and 1.28 mm thickness. Voltage probes were attached along the length of the rectangular Cd sample and along a radius of the Cd and Zn disk-shaped samples. Modulation fields up to 250 G peak-to-peak were obtained by exciting a coil wound on a pole cap of an electromagnet with the output of an audio oscillator amplified by a 150-W power amplifier. A PAR HR-8 phase-sensitive detector was used to measure the ac signals obtained on the samples which were immersed in liquid helium. A reference signal was obtained from a single insulated loop cemented to the face of each sample. This induced signal was amplified by a PAR CR-4 low-noise preamplifier after passing through a PAR model AM-1 impedance matching transformer. Various derivatives of the magnetomorphic oscillations were measured by tuning the HR-8 to harmonics of the oscillator output. Although the effect of varying the oscillator frequency was investigated, the data presented here were taken at 43 cps.

RESULTS AND DISCUSSION

Cadmium

Figure 1 shows the signal measured in the parallelepiped cadmium sample with $\mathbf{H}\|[0001]$. This signal is the output of the HR-8 which was tuned to 215 cps such that the data represent the fifth derivative of the magnetomorphic oscillation. The modulation field was 180 G peak-to-peak. The oscillation has a period of 92.6 G such that $P_0a = 143$ G mm (taking into account that the c axis of both Cd and Zn contract by 1.5% at helium temperatures).[5] This result may be compared to the value $P_0a = 135$ G mm reported by Grenier *et al.*[1] for the hole oscillations. The present data yield $\hbar^{-1}\partial S/\partial p_3 = 2.16$ Å^{-1} which may be compared to Grenier's value of 2.04 Å^{-1}. The short-period oscillations of Fig. 1 are evidently due to the hole arms of the second zone.

Figure 1 also shows data taken on the Cd disk. This represents a first derivative taken with a 43-cps modulation field of 100-G peak-to-peak amplitude. The magnetic field was parallel to [0001]. The period is 320 G yielding $\hbar^{-1}g = 1.44$ Å^{-1}. This may be compared to Grenier's value of 1.39 Å^{-1} and a free-electron value of 1.41 Å^{-1} for the apex of the cadmium lens. The lens oscillation was also observed in the cadmium parallelepiped where it exhibited a period of 382 G corresponding to $\hbar^{-1}g = 1.43$ Å^{-1}. Reference 3 reported a value of $P_0a = 569$ G mm for the apex of the cadmium lens. It has since been found that the magnetic sweep rates used in this reference must be corrected by 2.8%. The adjusted value for the data of Ref. 3 is $P_0a = 585$ G mm corresponding to $\hbar^{-1}g = 1.41$ Å^{-1}, which is identical with the free-electron value. However, measurements of the thickness a are accurate to $\pm 2\%$ so one may conclude from all of the available data that the upper curve in Fig. 1 represents lens oscillations and that $\hbar^{-1}g$ lies somewhere between 1.38 and 1.47 Å^{-1}. The voltage scales indicated in Fig. 1 should be noted. This laboratory has been unable to detect the hole-arm oscillations by dc techniques, and although the lens oscillations are easy to obtain, the amplitudes indicated in Fig. 1 are an order of magnitude larger than those obtained in a dc experiment with a current of 1 A in the crystal.

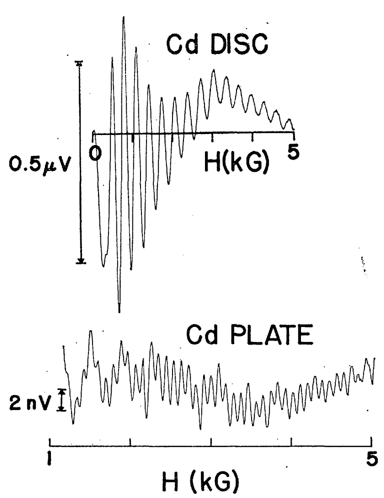

FIG. 1. Photographic reproductions of actual recorder traces with suitable scales superimposed. The long-period lens oscillation was observed in the Cd disk with phase-sensitive detection at the fundamental 43 cps. The short-period hole-arm oscillations were observed in the Cd plate with detection at 215 cps.

Zinc

Figure 2 shows the output of the HR-8 for the Zn disk. Modulation was at 43 cps with a 200-G peak-to-peak amplitude. Detection was also at 43 cps. When the magnetic field was parallel to [0001] Shubnikov–de Haas oscillations were excited, which tended to wash out the magnetomorphic oscillation. Figure 2 also indicates the effect of rotating the magnetic field 10° away from [0001]. The Shubnikov-de Haas oscillations no longer obscure the effect of interest, and approximately 16 periods are observed before the damping becomes overpowering. The period is 544 G corresponding to $\hbar^{-1}g = 1.68$ Å^{-1}. This value is 5% larger than the free-electron value of 1.60 Å^{-1}. The oscillation is evidently due to the lens in the third zone of Zn. Soffer[6] has reported observing a period of approximately 5 kG for this oscillation in a dc experiment on a flat

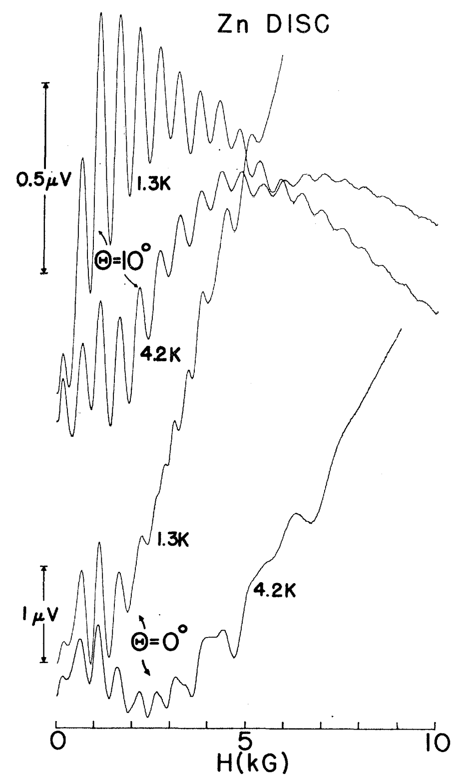

FIG. 2. Actual recorder traces of Zn data taken at both 1.3 and 4.2 K for H||[0001], $\theta=0°$, and H tilted 10° from [0001], $\theta=10°$. Shubnikov–de Haas oscillations appear in the $\theta=0°$ data, but disappear for $\theta=10°$. These data were taken with detection at 43 cps.

[5] R. W. Meyerhoff and J. F. Smith, J. Appl. Phys. **33**, 219 (1962); E. Grüneisen and E. Göens, Z. Physik **29**, 141 (1924).

[6] Stephen B. Soffer, Phys. Rev. **176**, 861 (1968).

plate 0.071 mm thick. The present data imply a period of approximately 9820 G for this thickness. As Soffer's data are exhibited over only 14 kG it is suggested that the oscillations shown in his Figs. 3 and 6 are only partial periods.

MODEL

An exact treatment of the problem of induced eddy currents in a finite cylinder due to a time-varying magnetic field directed along the axis of the cylinder is formidable, but the situation when the electronic mean free path is on the order of the cylinder dimensions is essentially isoluble. However, one may make some useful progress by assuming the following conditions are approximately met:

(a) The net magnetic field due to the superposition of the externally applied time-varying field and the eddy-current field is proportional to the applied time-varying field.

(b) The net field is uniform throughout the sample; i.e., the penetration depth is large compared to the cylinder length.

(c) The eddy-current density is uniform throughout the cylinder length at fixed radius.

(d) The time phases of the induced electric field and the resulting current density are equal and constant over the entire sample volume.

Let the external magnetic field H' be given by

$$H'=H+H_m \sin\omega t, \tag{2}$$

where H is a steady component. The resulting radial Hall field at radius R is given by

$$E\simeq\alpha(\rho_{21}/\rho_{11})H_m\omega R\cos\omega t, \tag{3}$$

where α is a factor which will be used to continually absorb proportionality factors as they arise. The quantities ρ_{21} and ρ_{11} are the Hall resistivity and transverse magnetoresistivity which are functions of H'. The time-dependent Hall voltage measured on probes at radii R_1 and R_2 is

$$V(t)\simeq\alpha H^{-2}H_m\omega(R_2^2-R_1^2)\cos(\omega t)\rho_{21}(H'). \tag{4}$$

Here the fact that ρ_{11} is essentially proportional to H^2 has been used. The component of ρ_{21} which is oscillatory in H is given by [3]

$$\tilde{\rho}_{21}\simeq\beta e^{-a/\lambda}\cos(2\pi H/P_0+\delta), \tag{5}$$

where β and δ are constants, a is the cylinder height, and λ is the bulk mean free path. This expression is derived in the limit $H\gg P_0$ and it is in this region one may expect the conditions assumed above to be valid. Expansion of $\tilde{\rho}_{21}(H')$ leads to an expression for the component of the Hall voltage which is oscillatory in H:

$$\tilde{V}\simeq\alpha e^{-a/\lambda}\omega(R_2^2-R_1^2)H^{-2}$$
$$\times[\cos(2\pi H/P_0+\delta)\sum_n \delta_0(n)J_n(2\pi H_m/P_0)\cos(n\omega t)$$
$$+\sin(2\pi H/P_0+\delta)\sum_n \delta_e(n)J_n(2\pi H_m/P_0)\sin(n\omega t)]. \tag{6}$$

Here $\delta_0(n)$ is zero for n even, one for n odd, and $\delta_e(n)$ is one for n even, zero for n odd. J_n is the nth-order Bessel function of the first kind.

DISCUSSION

Equation (6) shows that phase-sensitive detection at the nth harmonic of ω eliminates all but one term in the series expansion, such that one should observe $\tilde{V}$ to be periodic in H with period P_0. The predicted proportionality to ω was verified for ω in the range 20–150 cps. For ω above 150 cps, $\tilde{V}$ rose less steeply with ω. The proportionality to $R_2^2-R_1^2$ was found to be obeyed closely by using multiple probes placed at various radii. A log-log plot of amplitude against H showed qualitative agreement with the predicted factor of H^{-2}. The above model takes into account only the first-order eddy-current field. One may extend the calculation by computing the magnetic field due to this eddy-current distribution and then finding the resulting secondary eddy-current field etc., to generate a power series in ω. As discussed above, the measured proportionality of amplitude to ω $(R_2^2-R_1^2)$ is strong evidence that the leading term used in this model is predominant although the classical penetration depth in zinc at 43 cps varies from about 1 mm at $H=0$ kG to about 9 mm at $H=10$ kG. Therefore, Eq. (6) appears to account for the observed effect to a useful degree. The factor $\exp(-a/\lambda)$ accounts for the strong temperature dependence of the effect. It also may be used to extrapolate the present zinc data to the case of a sample 0.071 mm in thickness as was used in Soffer's experiment[6] where he observed an oscillation approximately 50 nv peak-to-peak (for an assumed 1 A in his crystal). Taking $\lambda\simeq1$ mm yields an extrapolated signal some 15 times as large for the eddy-current method. Thus one may obtain better than an order-of-magnitude gain over conventional techniques by this method. Further, one may work with substantially larger samples with the same degree of sensitivity.